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GRAVITATIONAL FIELD OF SPHERICAL BRANES

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The warped solution of Einstein's equations corresponding to the spherical brane in five-dimensional AdS is considered. This metric represents interiors of black holes on both sides of the brane and can provide gravitational trapping of physical fields on the shell. It is found the analytic form of the coordinate transformations from the Schwarzschild to co-moving frame that exists only in five dimensions. It is shown that in the static coordinates active gravitational mass of the spherical brane, in agreement with Tolman's formula, is negative, i.e. such objects are gravitationally repulsive.

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Motivated by string/M theory,¹ the AdS/CFT correspondence² and the hierarchy problem of particle physics,^{3,4,5} in recent years there was a lot of activity in brane-world models. In simplest brane-world scenario our universe is considered as a boundary of a 5D AdS bulk with induced Minkowski metric on the brane^{6,7,8,9}. There also exist various warped co-dimension-1 models with curved brane in AdS, e.g. the spatially flat cosmological brane-worlds,^{10,11,12,13,14,15} or the spherical static ones.^{16,17,18,19,20,21,22,23}

In thin-wall approximation the brane is considered as a hypersurface with δ -like singularity in its energy-momentum density

$$T_B^A = \delta\left(\frac{z}{\epsilon}\right) \text{diag}(T_0^0, T_1^1, T_2^2, T_3^3, 0) , \quad (1)$$

where ϵ is the length scale associated with the brane. Pressure towards the extra space-like dimension z is assumed to be zero. Capital Latin indices refer to 5D with the signature $(+ - - -)$. It is usually assumed that flat branes, which may also be considered as a domain walls, are described by the state equation²⁴

$$T_0^0 = -T_1^1 = -T_2^2 = -T_3^3 = \frac{\sigma}{\epsilon} = \text{const} , \quad (2)$$

where $\sigma > 0$ is the wall tension.

In the definition of active gravitational mass by Tolman's formula

$$m_{active} = \int \sqrt{g} d^4V (T_0^0 - T_1^1 - T_2^2 - T_3^3) = -2\sigma \int \sqrt{-g^{(4)}} d^3V , \quad (3)$$

where \sqrt{g} , d^4V are 5D and $\sqrt{-g^{(4)}}$, d^3V are 4D determinants and volume elements respectively, pressure terms act as a repulsive source of gravity and flat branes have negative gravitational mass.²⁴ Note that other topological objects also have peculiar gravitational properties. It was found that cosmic strings do not produce gravitational force on the surrounding matter locally, while global monopoles and global strings are repulsive.²⁴

Einstein's equations in five dimensions,

$$R_{AB} - \frac{1}{2} g_{AB} R = -\Lambda g_{AB} + \frac{1}{M^3} T_{AB} , \quad (4)$$

where M is the fundamental scale and $\Lambda > 0$ is bulk cosmological constant (corresponding to AdS), in the outer regions ($T_{AB} = 0$) to a flat brane have the well known solution^{6,7,8,9}

$$ds^2 = e^{-|z|/\epsilon} dl^2 - dz^2 . \quad (5)$$

The width of the brane ϵ relates to the brane tension, fundamental scale and bulk cosmological constant by the fine-tune conditions

$$\epsilon^2 = \frac{6}{\Lambda} = \frac{M^2}{6\sigma} . \quad (6)$$

In this so-called warped setup (5) the curvature is in the bulk but the brane is flat, i.e. the induced metric on the brane dl^2 is the ordinary 4D-Minkowski.

Curved branes in warped background are of much interest from the cosmological and gravitational point of view. In this paper we consider gravitational properties of a spherical 3D-brane embedded in a 5D AdS. It will be natural to think that this object, similarly to a flat brane, manifests gravitational repulsion, since it is assumed that flat and spherical domain walls obey the same state equations (2) (e.g. see Refs. 25 and 26). On the other hand, according to Birkhoff's theorem, the empty space surrounding any spherical body is described by the Schwarzschild metric, where the parameter corresponding to the mass of gravitating body is usually assumed to be positive. The disagreements in gravitational properties of planar and spherical domain walls were explained by instability of the latter,²⁶ or by the existence of a positive energy source stabilizing the bubble.²⁷ However, there still remain various paradoxes^{26,28,29} that can be solved only if bubbles with the state equation (2) are repulsive.

It is also possible that domain walls are not described by the state equation (2), since there exist several problems within models with large pressure (see recent papers^{30,31}). If nevertheless the state equation for a spherical object has the form (2), it is still possible to construct a model where repulsive bubbles are admissible. For example, one can consider reversion of time on the shell (analogously to what

happens between the upper and Cauchy radii in Reissner-Nordström metric),³² or use non-Minkowskian asymptotic.³³

The setup we consider here, 3D spherical brane in 5D AdS, is similar to static AdS black hole models^{16,17,18,19,20}. Unlike to these models, we do not restrict ourselves with the standard AdS-Schwarzschild solution (with the positive mass parameter), since in this case spherical domain walls do not exhibit gravitational trapping without extra sources.^{34,35} Instead we consider the metric *ansatz*

$$ds^2 = B^2(a) dt^2 - da^2 - A^2(a) d\Omega_3^2, \quad (7)$$

in the similar to (5) holographic-like frame (i.e. in Gaussian coordinates) and later check to what kind of geometry in co-moving coordinates it corresponds. In (7) the metric components $A(a)$ and $B(a)$ are functions of the 5D radial (holographic) coordinate a only, and

$$d\Omega_3^2 = d\kappa + \sin^2 \kappa d\theta^2 + \sin^2 \kappa \sin^2 \theta d\phi^2 \quad (8)$$

is the differential solid angle of the unit three-sphere with the three spherical angles κ, θ and ϕ .

In the Gaussian coordinates, the spherical brane is located at $a = R$. It is known that domain wall's metric with planar, or spherical symmetries is not in general static, but admits a de Sitter-like expansion.^{36,37,38,39,40,41} However, the problem we address in this paper is to find warped solution and active gravitational mass for spherical branes (as was mentioned above in a static coordinates they are expected to be gravitationally repulsive). So we consider static coordinates, where the radius of the bubble R is assumed to be constant.

For the *ansatz* (7) Einstein's field equations reduce to

$$\begin{aligned} \frac{A'^2}{A^2} + \frac{A''}{A} - \frac{1}{A^2} &= \frac{\Lambda}{3}, \\ \frac{A'^2}{A^2} + \frac{A'}{A} \frac{B'}{B} - \frac{1}{A^2} &= \frac{\Lambda}{3}, \\ \frac{A'^2}{A^2} + 2 \frac{A'}{A} \frac{B'}{B} + 2 \frac{A''}{A} + \frac{B''}{B} - \frac{1}{A^2} &= \Lambda, \end{aligned} \quad (9)$$

where primes denote derivatives with respect of a . From the first two equations, it follows that

$$B = C_1 A', \quad (10)$$

where C_1 is the integration constant. Then for the second unknown function we find

$$A^2 = C_2 e^{-a/\varepsilon} + C_3 e^{a/\varepsilon} - 2\varepsilon^2, \quad (11)$$

where C_2 and C_3 are additional integration constants. We also introduced the parameter

$$\varepsilon = \sqrt{\frac{3}{2\Lambda}}, \quad (12)$$

corresponding to the width of the brane.

Note that the solution represented by (10) and (11) is not new, it is just a special case (corresponding to the static brane) of cosmological brane-world solutions extensively discussed in the literature (see, e.g., Refs. 12-14). Our choice of boundary conditions is new. In (10) and (11) we want to choose integration constants C_1, C_2 and C_3 in order to receive the warped metric that on the surface of junction (on the spherical shell with the radius R) will coincide with the Minkowski metric. Recognizing the difficulty of handling thick walls within relativity we, as many other authors, will use thin-wall formalism.^{25,42} To obtain warped metric for the shell, we need decreasing functions A^2 and B^2 obeying the boundary conditions

$$A^2(a = R) = R^2, \quad B^2(a = R) = 1. \quad (13)$$

In general, in the first expression we can multiply R^2 by some constant that will correspond to a 3D shell with a solid angle deficit.

The boundary conditions (13) fix the values of integration constants for the outer region to the shell

$$\begin{aligned} C_1^+ &= \frac{2\varepsilon R}{R^2 + 2\varepsilon^2}, \\ C_2^+ &= (R^2 + 2\varepsilon^2)e^{R/\varepsilon}, \\ C_3^+ &= 0. \end{aligned} \quad (14)$$

Thus the metric functions outside the shell ($a > R$) are

$$\begin{aligned} A_+^2 &= (R^2 + 2\varepsilon^2)e^{-(a-R)/\varepsilon} - 2\varepsilon^2, \\ B_+^2 &= \frac{R^2}{A_+^2}e^{-2(a-R)/\varepsilon}. \end{aligned} \quad (15)$$

Note that because of warped background the usual cancellation of Newton's gravitational potential inside a spherical shell does not occur and we can assume the Z_2 -symmetry between the two sides of the shell. Then integration constants inside can be chosen as

$$\begin{aligned} C_1^- &= \frac{2\varepsilon R}{R^2 + 2\varepsilon^2}, \\ C_2^- &= 0, \\ C_3^- &= (R^2 + 2\varepsilon^2)e^{-R/\varepsilon}, \end{aligned} \quad (16)$$

and the metric functions in the interior region ($a < R$) have a form similar to (15)

$$\begin{aligned} A_-^2 &= (R^2 + 2\varepsilon^2)e^{(a-R)/\varepsilon} - 2\varepsilon^2, \\ B_-^2 &= \frac{R^2}{A_-^2}e^{2(a-R)/\varepsilon}. \end{aligned} \quad (17)$$

One can choose another solution inside the shell, e.g. 5D Minkowski, or Minkowski-AdS. We don't consider these possibilities, since in this paper we want to show

existence of warping for a spherical brane that approximates the flat one (5) when the radius of the shell is large.

Note that our metric (7), similarly to the case of asymmetrically warped space-times models,^{21,22,23} contains two different warp factors A_{\pm}^2 and B_{\pm}^2 , one associated with time and another with 3D-space. This violates Lorentz invariance^{43,44} and in general light speed on the shell and in the bulk will be different.⁴⁵

The metric functions $A_{\pm}(a)$ in (15), (17) become zero at some distance d . Thus space-time of the shell with Z_2 symmetry has singularities (horizons) in both, interior and exterior regions, which can be cancelled by the introduction of some sources in the bulk. We can estimate the distance to these singularities

$$d = \varepsilon \ln \left(\frac{R^2 + 2\varepsilon^2}{2\varepsilon^2} \right) \sim 100 \varepsilon \quad (18)$$

if we use, for example, the values of the radius of curvature of our universe $R \sim 10^{28} \text{ cm}$ and experimentally acceptable width of the brane $\varepsilon \sim 10^{-3} \text{ cm}$.

The second metric functions B_{\pm}^2 ('potential') in (15), (17) exhibits slightly different behavior. It decreases from the brane location to its minimal value

$$B_{min}^2(a - R = d) = \frac{8\varepsilon^2 R^2}{(R^2 + 2\varepsilon^2)^2}, \quad (19)$$

(that is a very small number) at the distance $d - \varepsilon \ln 2$. Then it starts to grow up and approaches infinity at d .

Using the expressions (15) and (17) our *ansatz* (7) can be rewritten in a form similar to (5) with the common warp factor for 4D part of the metric

$$ds_{\pm}^2 = \left[\frac{R^2 + 2\varepsilon^2}{a^2} e^{\mp(a-R)/\varepsilon} - \frac{2\varepsilon^2}{a^2} \right] [U_{\pm}^2(a) dt^2 - a^2 d\Omega_3^2] - da^2, \quad (20)$$

where the values of the functions

$$U(a)_{\pm} = \frac{R a e^{\mp(a-R)/\varepsilon}}{(R^2 + 2\varepsilon^2) e^{\mp(a-R)/\varepsilon} - 2\varepsilon^2}, \quad (21)$$

close to the brane ($a \approx R$) are approximately equal to 1. As it was expected, in the large-shell limit, $a \sim R \gg \varepsilon$, singularities of the metric (20) are shifted to the infinity and (20) transforms to the metric of a flat brane (5).

The determinant in our *ansatz* (7) is given by

$$\sqrt{g} = \sqrt{-g^{(4)}} \frac{A^3(a) B(a)}{R^3}, \quad (22)$$

where $\sqrt{-g^{(4)}}$ is the 4D determinant on the shell. Localization of the 4D spin-2 graviton on the spherical brane requires the integral of the gravitation part of the action,

$$S = \int d^5 x \sqrt{g} \frac{M^3}{2} R, \quad (23)$$

over a to be convergent. If we neglect the effects of $U(a)_\pm$ our metric (20) will have the conformal structure similar to (5) and only the integral of the determinant (22) will be nontrivial. In this case for the effective Planck's scale on the shell we find

$$m_{pl}^2 \approx \frac{M^3}{R^3} \int da A^3 B = \frac{4\varepsilon M^3}{R^2(R^2 + 2\varepsilon^2)} \int_0^R dA A^3 = \frac{\varepsilon R^2 M^3}{(R^2 + 2\varepsilon^2)} \approx \varepsilon M^3, \quad (24)$$

which is the standard expression for the large extra dimensional models.^{3,4,5} In the case of shell with a deficit angle in (24) a new parameter will appear and one will be able to explain large hierarchy even in 5D.

Now we can translate our solutions (15), (17) into static co-moving coordinates by introducing the new 'radial' coordinates

$$r_\pm = A_\pm, \quad (25)$$

which are connected to the proper radius a by

$$a = R + \varepsilon \ln \left(\frac{R^2 + 2\varepsilon^2}{r_\pm^2 + 2\varepsilon^2} \right) = R + \varepsilon \ln \left(\frac{r_\pm^2 + 2\varepsilon^2}{R^2 + 2\varepsilon^2} \right). \quad (26)$$

Then (7) transforms to Schwarzschild's coordinates

$$ds^2 = D(r_\pm) dt^2 - \frac{dr_\pm^2}{D(r_\pm)} - r_\pm^2 d\Omega_3^2, \quad (27)$$

where the only independent metric function has the form

$$D(r_\pm) = 1 + \frac{\varepsilon^2}{r_\pm^2} + \frac{r_\pm^2}{4\varepsilon^2}. \quad (28)$$

We see that horizons of the metric (20) in the new coordinates correspond to the central singularities of the Schwarzschild metrics at $r_\pm = 0$. So spherical brane in our setup can be imagined as the common boundary (event horizon) of two black holes situated at different sides of the shell.

Recalling the definition of the brane width (12), we recognize that the last term in (28),

$$\frac{r_\pm^2}{4\varepsilon^2} = \frac{\Lambda_\pm}{6} r_\pm^2, \quad (29)$$

is the standard term with the cosmological constant in 5D AdS-Schwarzschild metric.

Note that the value and the sign of the second term should represent the gravitational potential in five dimensions. Because of the positive value of the potential, we conclude that warped spherical brane in AdS should have negative active gravitational mass equal to

$$m_{brane} = -\frac{3M^3}{2\Lambda}. \quad (30)$$

This means that static spherical brane with warped geometry should be gravitationally repulsive in agreement with Tolman's formula (3). Using the formulae (3)

and (22), and the expression for the volume of 3D-sphere ($V^3 = 2\pi^2 R^3$), from (30) one can find that the brane tension is related to the shell radius, fundamental scale and bulk cosmological constant as

$$\sigma = \frac{3M^3}{8\pi^2 R^3 \Lambda} . \quad (31)$$

This is analogue to the fine-tuning condition (6) in the flat brane model (5).

In this paper the warped solutions (15), (17) of Einstein's equations corresponding to the spherical brane in 5D AdS is considered. These solutions in Schwarzschild's coordinates represent interiors of black holes placed at both sides of the brane. So in realistic models one needs some sources in the bulk to cancel these singularities and also to stabilize the shell. It is shown that spherical branes in static coordinates are gravitationally repulsive in agreement with the estimations of Tolman's formula (3). Existence of warping in our model means that matter fields can be trapped gravitationally by spherical branes, similarly to the case of flat ones. Note that our analytic solutions, and simple transformation to co-moving coordinates (26), takes place only in five dimensions, where the orders of the second and third terms in the Schwarzschild metric function (28) (corresponding to the gravitational potential and the cosmological constant respectively) coincide.

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